Channel Equalization For ISI Cancellation

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***Abstract*— This paper deals with a receiver scheme where adaptive equalization and channel sounding are jointly optimized in an iterative process.We discuss an overview of important channel equalization tasks such as Zero Forcing Algorithm, LMMSE filter and The Gradient (LMS) Algorithm process for inter symbol interference cancellation. This receiver scheme is well suited for transmissions over a frequency-selective channel with large delay spread and for high spectral efficiency modulations.**

1. INTRODUCTION

An Equalizer is a compensator for Channel Distortion. For communication channels in which the channel characteristics are unknown or time-varying, optimum transmit and receive filters cannot be designed directly. For such channels, an equalizer is needed to compensate for the ISI created by the distortion in the channel. There are three types of equalization methods commonly used:

- Maximum Likelihood (ML) Sequence
  Detection - Optimal, but Impractical.
- Linear Equalization - Suboptimal, but simple.
- Non-Linear Equalization (DFE)- for severe ISI channels

Linear Equalizers are simple to implement and are highly effective in channels where is the ISI is not severe (like the wire line telephone channel). Most linear equalizers are implemented as a linear transversal filter, shown in Figure (1). where, the number of equalizer taps is 2M + 1 and T is the symbol duration. If Y (t) is the input to the equalizer, then the output of the equalizer is given by $Yeq(t) = \sum_{-M}^{i=M} \omega i Y(t - iT)$ where, ωi are the complex equalizer tap weights selected based on some optimization criterion.

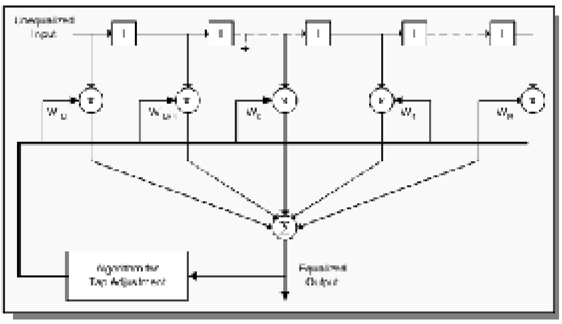

Figure 1. Linear Transversal Equalizer

1.1 CRITERION FOR OPTIMIZATION

Two criteria are commonly used for optimizing the equalizer tap weights.

- Peak Distortion Criterion - leading to the Zero-Forcing Equalizer.
- Mean Square Error (MSE) Criterion - leading to the LMMSE equalizer and The Gradient (LMS) algorithm.

1.2 WEIGHT ADAPTATION

Two criteria are commonly used for optimizing the equalizer tap weights

• Preset Equalizers, used for channels in which the frequency response characteristics are unknown, but invariant (such as the telephone channel). The Weights are calculated only once (in the beginning of the session) are not varied during the session.

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• Adaptive Equalizers, used for channels in which the frequency response is time-variant. These equalizers are capable of tracking a slowly time-varying channel by updating their parameters on a periodic basis.

2. ZERO FORCING EQUALIZER EQUALIZER

The Zero-Forcing Equalizer belongs to the class of preset linear equalizers and it uses the Peak Distortion Criterion to evaluate the equalizer tap weights. Consider the communication system

block diagram (with an equalizer) given in Figure (2).

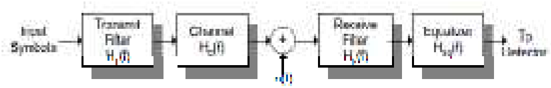

Figure (2) communication system block diagram (with equalizer)
Assuming that a raised-cosine pulse shape is used, the condition for zero-ISI is given by,

$$H_T(f)H_C(f)H_R(f)H_{Eq}(f) = H_{rc}(f)$$

We have by definition $H_T(f)H_R(f) = H_{RC}(f)$
So the value of $HEq(f)$ that compensates for the channel distortion $H_C(f)$ is given by,

$$H_{Eq}(f) = \frac{1}{H_C(f)}$$

This equalizer is also called the inverse channel equalizer. The discrete time version of the above equation with a sampling rate equal to the symbol-duration T, is given $1 = H_{Eq}(f)H_C(f)$
Or, in the discrete-time domain, we have,

$$\sum_{i=-\infty}^{+\infty} \omega_j h(n-j) = p_{eq}(n) = \begin{cases} 1, if & n = 0 \\ 0, if & n \neq 0 \end{cases}$$

Where, h(n) is the (discrete-time) impulse response of the channel and pEq(t) is the response after equalization. Since, the filter forces due the ISI to zero, it is also called the Zero-Forcing Equalizer. In practice, since the ISI caused by the channel is limited to a finite number of symbols on either side of the desired symbol, it required to force the ISI to zero only at those sampling instants. The result is a finite duration (FIR) traversal filter as shown in fig(1).

2.1 DERIVATION OF THE FILTER COEFFICIENTS

Let $p_r(t)$ represents the unequalized pulse corresponding to the cascade of $H_T(f)$, $H_C(f)$ $H_R(f)$ and the input (test) pulse $p(t)$ into the channel. Let the length of the transversal filter be 2M + 1.The equalized output pulse is given by $p_{eq}(t) = \sum_{i=-M}^{M} \omega_i p_r(t - iT)$
The Zero-Forcing condition is now applied to the samples of pEq(t) taken at sampling times t = mT.

$$p_{eq}(mT) = \sum_{i=-M}^{+M} \omega p_r((m-i)T = \begin{cases} 1, if & m = 0 \\ 0, if & m = 0, \pm1, \pm2, \ldots\ldots, \pm M \end{cases}$$

which results in a set of 2M + 1 simultaneous equations, whose solution is given by,

$$\begin{pmatrix} \omega_{-M} \\ . \\ . \\ \omega_0 \\ . \\ . \\ \omega_M \end{pmatrix} = \begin{pmatrix} p_r(0) & p_r(-1) & . & . & . & p_r(-2M) \\ p_r(1) & p_r(0) & . & . & . & p_r(-2M+1) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ p_r(2M-1) & . & . & . & p_r(0) & p_r(-1) \\ p_r(2M) & . & . & . & p_r(1) & p_r(0) \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ . \\ . \\ 1 \\ . \\ . \\ 0 \end{pmatrix}$$

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It is important to note that the equalizer designed using the above equation does not completely eliminate ISI, as it has a finite length. The remaining or residual ISI can be reduced by increasing M, but a larger M will increase the complexity of both the equalizer design and implementation. As M →∞, ISI is completely eliminated.

2.2 DISADVANTAGE

The Zero-Forcing Equalizer design does not take into account the effect of additive noise. Since the equalizer frequency response is approximately the inverse of the channel's
frequency response (which is usually low pass), it will cause a significant enhancement in the noise power at high frequencies. So, an equalizer designed with both Noise and ISI taken into account will offer a much better performance than the Zero-Forcing Equalizer.

3. LINEAR MINIMUM MEAN SQUARED ERROR (LMMSE.)

Filter The Zero-Forcing Equalizer has a severe drawback due to its noise performance. The LMMSE filter overcomes this drawback by relaxing the zero-ISI condition and selecting the equalizer characteristic such that the combined power in the ISI and the additive noise at the equalizer output is minimized.

3.1 BASIC ASSUMPTIONS

The following are assumed in the derivation of the LMMSE Equalizer.

- The input symbols are temporally uncorrelated, $E\{A_n A_{n+j}\} = \delta(j)$
- The input symbols are uncorrelated with noise.

3.2 DERIVATION OF THE EQUALIZER TAP WEIGHTS.

Consider the input to the equalizer

$$Y(n) = \sum_{k=-\infty}^{\infty} A_k p_r(n-k) + N(n)$$

And the equalizer output,

$$Y_{eq}(n) = \sum_{i=-M}^{M} \omega_i Y(n-i)$$

The Equalizer output at the sampling times are given by,

$$Y_{eq}(n) = \sum_{i=-M}^{M} \omega_i \left[\sum_{k=-\infty}^{\infty} A_k p_r(n-i-k) + n(n-i)\right]$$

The quantity to be minimized the mean squared error (MSE) given by,

$$MSE = E\left\{\left(A_n - Y_{eq}(n)\right)^2\right\}$$

Using the above equations, we can re-write the expression for the MSE as,

$$MSE = E\{A_n^2\} - 2\sum_{i=-M}^{M} \omega_i E\{A_n Y(n-i)\} + E\left\{\left(\sum_{i=-M}^{M} \omega_i Y(n-i)\right)^2\right\}$$

Further simplification to the above equation is possible by using the following facts $E\{A_n^2\} = \sigma_A^2$ (by definition).

$E\{A_n Y(n-i)\} = \sigma_A^2 \; p_r(-i)$ (using the basic assumptions).

$E\{Y(n-i)Y(n-j)\} = \sigma_A^2 \; \gamma(i,j) + \sigma_N^2 \; p(i,j)$. (using the basic assumptions).

Where,

$\gamma(i,j) = \sum_{l=-\infty}^{\infty} p_r(l-i)\, p_r(l-j)$ represents the auto-correlation of the unequalized pulse values.

$p(i,j) = E\{n(n-i)n(n-j)\}$)} represents the auto-correlation of the receiver input noise.

σ_A^2 represents the Average Signal Power.

σ_N^2 represents the Average Noise Power.

The aboveequation can be converted into a compact matrix notation using these following definitions

$Pr = [p_r(M),\ldots, p_r(0),\ldots, p_r(-M)]^T$ are the received pulse values.

Γ is a matrix whose (i, j)th element is $\gamma(i,j) + (\sigma_N^2 / \sigma_A)\, p(i-j)$

$w = [\omega_{-M},\ldots, \omega_0,\ldots, \omega_M]$ represents the equalizer tap-weights.

The final expression for the MSE is,

$MSE = \sigma_A^2 (1 - 2\,P_r^T) w + w^T \Gamma w$

Differentiating the above equation with respect to the equalizer tap weights ωi and setting the resulting expressions to zero, we arrive at a set of simultaneous equations given by,

$\Pr = \Gamma w$

Note that if the input noise is uncorrelated (or white) the above equation becomes,

$$\Pr = (R + \frac{\sigma_N^2}{\sigma_A^2} I) w$$

Where, R is the auto-correlation matrix of the unequalized pulse values and I is a identity matrix. The equalizer tap-weights can be computed form the above equation as :

$$w = (R + \frac{\sigma_N^2}{\sigma_A^2} I)^{-1} \Pr$$

Recognizing the fact that $\sigma_A^2 / \sigma_N^2 = SNR$,

The above equation can be used to visualize the trade-off between Noise and ISI in the design of the LMMSE equalizer. If the receiver is operating in an almost noise-free condition (SNR →∞), the second term in the matrix goes to zero and the solution approaches the zero-forcing equalizer solution. In all other cases, the noise term is weighed appropriately (based on the SNR).

3.3 Existence Of And Optimal Solution

The existence of an optimal solutions basically rests with the ability to invert the Covariance () matrix. In general, the Coavariance matrix is symmetric non-negative definite. The definiteness of the matrix is strengthened by the the additive white noise component that provides a strong diagnal component, thus preventing singularity.

3.4 Residual MSE

The Residual MSE after the equalization can be computed using the expression,

$MSE = \sigma_A^2 (1 - P_r^T \Gamma \Pr)$

4. The Gradient (LMS) algorithm

The LMS or Gradient algorithm is used to implement adaptive equalization. It is a stochastic gradient optimization algorithm based on a traditional optimization technique called the Method of Steepest Descent. This algorithm takes into advantage the following facts :

• The Mean square error (MSE) surface when plotted against the filter coeffcients is a quadratic,bowl-shaped one with an unique minimum.

• The Gradient of a function always points "uphill", i.e, towards the maximum of the function.(Conversely, the negative of the gradient is a vector quantity always pointing towards the minimum).

4.1 The Method Of Steepest Descent

In the Steepest Descent Optimization method, the weight vector is made to "evolve" in the direction of the negative gradient of the MSE :

$w[n+1] = w[n] + \frac{\Delta}{2} \left[\left(-\nabla \left(E \left\{ \in^2 [n] \right\} \right) \right) \right]$

Where n reprsents the iteration number, $\in [n]$ is the error between the actual and the desired outputs in the nth iteration and Δ is the step-size. The main disadvantage of this method is the calculation of the actual gradient, which involves ensemble averages not readily available in real time.

4.2 The LMS Simplification

The LMS algorithm is bascially asimplification of the Method of Steepest Descent, where instantaneous values are used instead of actual (ensemble averaged) values:

$$w[n+1] = w + \Delta \in [n] * Y_i[n]$$

Where

$Y_i[n] = [Y((n-M)T), \ldots, Y(0), \ldots, Y((n+M)T)]^T$ represents the tap-inputs at the time instant (or iteration) n. The error [n] is computed from the equalized output using either a training sequence or the decoded output as reference. $\in [n] = A_n - y_{eq}[n]$

Where yeq[n] is the equalized output.

4.3 THE ALGORITHM

Using the Expressions for the error (29) and the output in the above equation(28), we get steps the LMS algorithm as :

$$y_{eq}[n] = w[n]^H Y_i[n]$$
$$\in [n] = A_n - y_{eq}[n]$$
$$w[n+1] = w[n] + \Delta \in^* [n] Y_i[n]$$

The block diagram for implementing this algorithm is shown in fig(7).

4.4 STABILITY

It has been shown that starting with an arbitrary initial weight vector, the LMS algorithm will converge and stay stable as long as the value of Δ is chosen as per the following rule:

$$0 < \Delta < \frac{1}{\lambda_{max}}$$

Where λ_{max} is the maximum eigenvalue (or trace) of the Covariance Matrix, Δ.

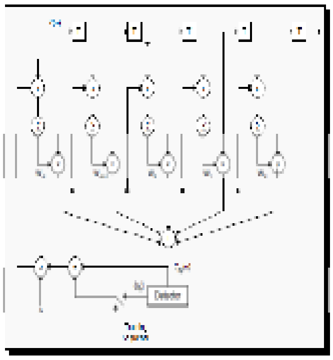

Figure-7 LMS Adaptive Equalizer

4.5 CONVERGENCE

Within the margin given by equation (33), the larger the value of Δ, the faster the convergence but the lesser the stability around the minimum value. On the other hand, the smaller the value of Δ, the slower the convergence but the algorithm will be more stable around the optimum value. An good choice for Δ in the case in which the SNR is known is :

$$\Delta = \frac{0.2}{(2M+1)(S+N)}$$

RESULT ANALYSIS

5.1. ZERO FORCING EQUALIZER

5.1.1. CHANNEL SOUNDING.

The Impulse response of the given channel (4th order Butterworth

m	$p_r(mTb)$
-6	0
-5	0
-4	0
-3	0
-2	0
-1	0.1234
0	0.8255
1	0.1131
2	-0.0912
3	0.0358
4	-0.0057
5	-0.0023
6	0.0021

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The input pulse in the channel p(t) and the output pulse $p_r(t)$ are plotted in figure(3). The ISI due to the channel and the channel delay are evident from the plots. It fact, the optimal sampling point of the pulse was computed to be 27 samples with respect to a common time-origin. Using the

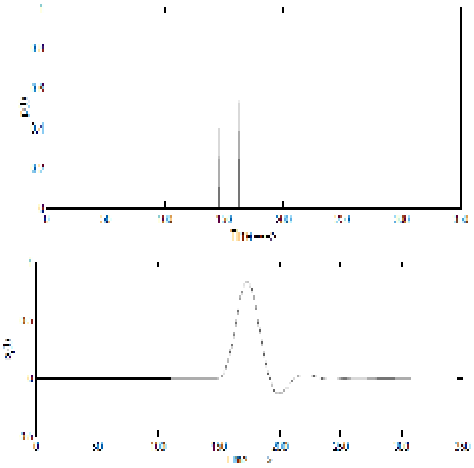

Figure-3 Channel sounding tabulated values in the above equation, we get,

$$\begin{pmatrix} w_{-3} \\ w_{-2} \\ w_{-1} \\ w_0 \\ w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 0.8256 & 0.1234 & 0 & 0 & 0 & 0 & 0 \\ 0.1131 & 0.8255 & 0.1234 & 0 & 0 & 0 & 0 \\ -0.0912 & 0.1131 & 0.8255 & 0.1234 & 0 & 0 & 0 \\ 0.0458 & -0.0912 & 0.1131 & 0.8255 & 0.1234 & 0 & 0 \\ 0.0077 & 0.0458 & 0.0912 & 0.1131 & 0.8255 & 0.1234 & 0 \\ -0.0023 & -0.0027 & 0.0458 & -0.0912 & 0.1131 & 0.8255 & 0.1234 \\ 0.0021 & -0.0023 & -0.0057 & 0.0503 & -0.0912 & 0.1131 & 0.8255 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The Zero-Forcing Equalizer Weights are computed as,

$$\begin{pmatrix} w_{-3} \\ w_{-2} \\ w_{-1} \\ w_0 \\ w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} -0.0045 \\ 0.0298 \\ -0.1952 \\ 1.2755 \\ -0.2271 \\ 0.1970 \\ -0.1087 \end{pmatrix}$$

The eye-diagrams at the equalizer input and output are plotted in figures (4) and (5).

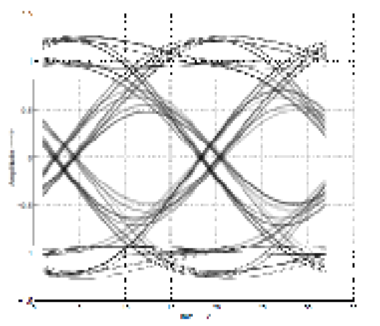

Figure-4 Eye diagram at Equalizer input

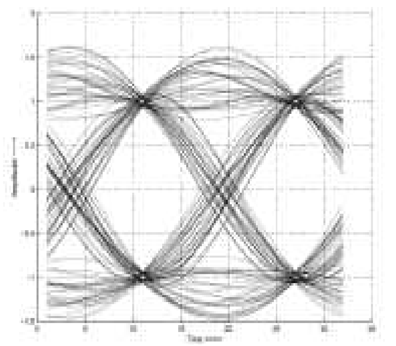

Figure-5 Eye diagram at Equalizer output

It is clear from the eye-diagram plots that the equalizer has indeed reduced the ISI by a considerable amount and the eye-opening has increased. A comparision of the original and the equalized pulses is shown in figure(6) to illustrate the reduction of ISI.

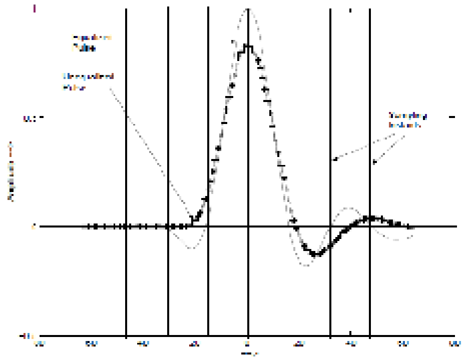

Figure 6. Comparision of the Unequalized and Equalized Pulses

5.1.2 MSE AT SAMPLING TIMES.

Ignoring the first 32 bits from the equalizer input and output, the MSE was estimated as :

MSE (@ input) = 0.0680
MSE (@ output) = 0.0023

5.2. LMMSE EQUALIZER

Using the data collected by Channel sounding, the R matrix was estimated as,

$$\mathbf{R} = \begin{pmatrix} 0.7192 & 0.1815 & -0.0569 & 0.0180 & -0.0008 & -0.0023 & 0.0013 \\ 0.1815 & 0.7192 & 0.1815 & -0.0569 & 0.0180 & -0.0008 & -0.0023 \\ -0.0569 & 0.1815 & 0.7192 & 0.1815 & -0.0569 & 0.0180 & -0.0008 \\ 0.0180 & -0.0569 & 0.1815 & 0.7192 & 0.1815 & -0.0569 & 0.0180 \\ -0.0008 & 0.0180 & -0.0569 & 0.1815 & 0.7192 & 0.1815 & -0.0569 \\ -0.0023 & -0.0008 & 0.0180 & -0.0569 & 0.1815 & 0.7192 & 0.1815 \\ 0.0013 & -0.0023 & -0.0008 & 0.0180 & -0.0569 & 0.1815 & 0.7192 \end{pmatrix}$$

The value of the signal-to-noise ratio, was computed from the value of Eb/No as,

$$SNR = \frac{\sigma^2{}_A}{\sigma^2{}_N} = \frac{1}{8}\left(\frac{E_b}{N_0}\right)$$

The tap-weights were computed directlyand the results are given in the following table

m	$w_m(E_b/N_0 = \infty)$	$w_m(E_b/N_0 = 10)$	$w_m(E_b/N_0 = 2)$	$w_m(E_b/N_0 = 1)$
-3	-0.0039	0.0232	0.0079	0.0042
-2	0.0283	-0.0149	-0.0153	-0.0102
-1	-0.1852	0.0167	0.0153	0.0113
0	1.2714	0.5384	0.1732	0.0941
1	-0.2194	0.0165	0.0197	0.0123
2	0.1830	0.0191	0.0013	0.0003
3	-0.0959	-0.0061	-0.0005	-0.0001

The MSE was computed as and the fact that $\sigma^2{}_A = 16$

5.3 THE GRADIENT (LMS) ALGORITHM

The Gradient algorithm was setup as shown in the figure (7) and the following results were obtained. The number of samples simulated were 5120 samples and the MSE was calculated for the last 2560 samples. (If only 512 samples were used, the results obtained were not accurate, i.e, either the algorithm did not converge or the residual MSE was too large for the result to be resonably accurate). The value of Δ was set based on equation (34).

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5.3.1. FINAL WEIGHT VECTORS.

The final equalizer weight vectors are tabulated in Table 2.

m	$w_{eq}(E_b/N_0=\infty)$	$w_{eq}(E_b/N_0=10)$	$w_{eq}(E_b/N_0=2)$	$w_{eq}(E_b/N_0=1)$
-3	-0.0085	0.0454	0.0158	0.0189
-2	0.0200	-0.0409	-0.0684	-0.0156
-1	-0.1989	0.0148	0.0658	-0.0514
0	1.2736	0.5279	0.1741	0.1010
1	-0.2300	0.0163	0.0129	-0.0115
2	0.1865	0.0310	-0.0302	0.0134
3	-0.0081	-0.0143	0.0152	-0.0025

Table-2 Equalizer Weights

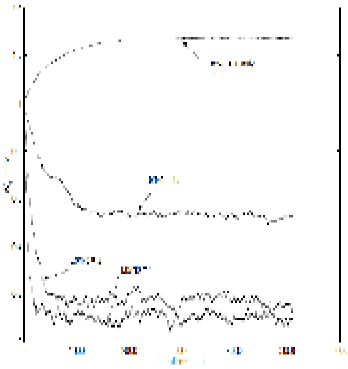

Figure 8. Weight Vector Convergence

It can be noted that the equalizer weights after the gradient algorithm has converged are similar to the weights computed for the LMMSE equalizer.

5.3.2 MSE.

The MSE was computed for the last 2560 samples as:

$$MSE=\begin{cases}0.9441, & E_b/N_0=1\\ 0.8620, & E_b/N_0=2\\ 0.5518, & E_b/N_0=10\end{cases}$$

The MSE after the gradient algorithm has converged is approximately the same as the MSE found for the LMMSE equalizer.

5.3.3. WEIGHT VECTOR CONVERGENCE.

The figure (8) illustrates the weight vector convergence.

5.4. CONCLUSIONS.

Linear equalizers are widely used for high-speed modems that transmit data over telephone channels.The LMMSE-based channel equalization has been discussed in this paper. We have found that the eye-diagram plots that the equalizer has indeed reduced the ISI by a considerable amount and the eye-opening has increased. The effectiveness of the proposed method has been demonstrated through computer simulations.

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