FINITE ELEMENT BASED VIBRATION ANALYSIS OF A CRACKED CIRCULAR BEAM

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Abstract: The present day structures and machineries are designed based on optimizing of multi-objectives such as maximum strength, maximum life, minimum weight and minimum cost. Due to this they are flexible and allow having a very high level of stresses. This leads to development of cracks in their elements. Many engineering structures may have structural defects such as cracks due to long-term service. So it is very much essential to know the property of structures and response of such structures in various cases. In order to make the structural element smart, adaptive and self-controlling piezoelectric patches are generally used. In this article the natural frequencies and mode shapes of an uncracked and cracked Timoshenko beam is studied by using finite element method (FEM) and MATLAB programme. The effect of piezoelectric patches on natural frequencies of the uncracked and cracked Timoshenko beam along with piezoelectric bimorph is studied for electro-mechanical validation.

Keywords: Cracked beam, Timoshenko beam, finite element method, piezoelectricity, bimorph beam**.**

1. Introduction

Presence of crack in a structural member is a serious threat to the performance of the structure. The effects of crack on the dynamic behavior of the structural elements have been the subject of several investigations for the last few decades. Due to the existence of such cracks the frequencies of natural vibration, amplitudes of forced vibration, and areas of dynamic stability change. In order to identify the magnitude and location of the crack, analysis of these changes is essential. The information from the analysis enables one to determine the degree of sustainability of the structural element and the whole structure. The Beams are one of the most commonly used elements in structures and machines, and fatigue cracks are the main cause of beams failure.

Papadopoulos and Dimarogonas [1] revealed the introduction of local flexibility due to presence of transverse crack in a structural member whose dimension depends on the number of degrees of freedom considered. It has been observed that the local flexibility matrix is mainly appropriate for the analysis of a cracked beam if one employs an analytical method by solving the differential equations piece wisely [2]. One way to detect cracks on structures is to employ modal testing in which changes in modal parameters such as variations in frequencies and mode shapes are used to detect damage. The detection of structural damage through changes in frequencies was discussed by Salawu [3]. Moreover, Dilena and Morassi [4] proposed damage identification based on changes in the nodes of mode shapes. They demonstrated that appropriate use of resonances and anti-resonances can be used to avoid the non-uniqueness of damage location for symmetrical beams.

Identification of cracks in beam structures using Timoshenko and Euler beam formulation has been studied by Swamidas*et al*. [5]. In their paper Timoshenko and Euler beam formulations have been used to estimate the influence of crack size and location on natural frequencies of cracked beam. Frequency contour method has been used to identify the crack size and location properly. Ali *et al*.[6] have studied the free vibration analysis of a cantilever beam. It has been observed that the presence of crack in the beam, will affect the natural frequency. The magnitude for the change of natural frequency depends on the change of (number, depth and location) for the crack. Also the change of dynamic property will affect on stiffness and dynamic behavior. Shin *et al*. [7] have developed a method to find the lowest four natural frequencies of the cracked structure by finite element method. They have obtained the approximate crack location by using Armon's Rank-ordering method that uses the above four natural frequencies. A method for shaft crack detection have proposed by Xiang *et al*. [8] which is based on combination of wave-let based elements and genetic algorithm. The experimental investigations of the effects of cracks and damages on the structures have been reported by Owolabi et al. [9]. Al-Qaisia*et al*. [10] have utilized the reduction of Eigen frequencies and sensitivity analysis to localize a crack in a non-rotating shaft coupled to an elastic foundation. The shaft was modeled by the finite element method and coupled to an experimentally identified foundation model. Sahin*et al*. [11] have introduced different damage scenarios by reducing the local thickness of the selected elements at different locations along with finite element model (FEM) for quantification and localization of damage in beam-like structures in their research. Nahvi*et al*. [12] have developed an analytical, as well as experimental approach to the crack detection in cantilever beams by vibration analysis.

E.Viola*et al*. [13] have used a finite element method for static and dynamic analysis of a cracked prismatic beam on the basis of Hamilton's principle. The crack section was modeled as an elastic hinge by considering fracture mechanics theory. Kisa*et al*. [14] have developed the component mode synthesis technique along with finite element method for free vibration analysis of uniform and stepped cracked beam with circular cross section. The finite element analysis of a cracked cantilever beam and the relation between the modal natural frequencies with crack depth, modal natural frequency with crack location has been studied by Sutar [15]. Only single crack at different depth and at different location are evaluated. The analysis reveals a relationship between crack depth and modal natural frequency. Zheng [16] has described an overall flexibility matrix instead of local flexibility matrix in order to find out the total flexibility matrix and the stiffness matrix of the cracked beam. It has been observed that the consideration of „overall additional flexibility matrix", due to the presence of the crack, can indeed give more accurate results than those obtained from using the local flexibility matrix. The overall additional flexibility matrix parameters are computed by 128-point (1D) and 128 ×128(2D) Gauss quadrature and then further best fitted using the least-squares method. The best-fitted formulas agree very well with the numerical integration results [17]. After getting the stiffness matrix of a cracked beam element standard FEM procedure can be followed, which will lead to a generalized eigen-value problem and thus the natural frequencies can be obtained.

2. Mathematical modeling of a circular cracked beam

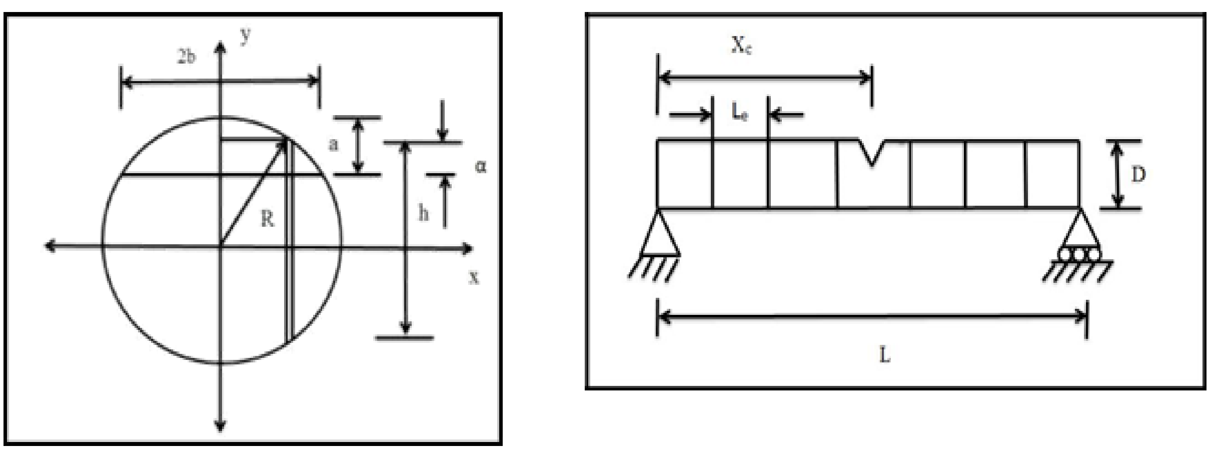

Figure 1. Simply supported beam with crack subjected to shear force and bending moment.

Fig.1 shows a simply supported beam of circular cross-section having diameter D with a single transverse crack with constant depth a. The crack is at a distance of X_c from the left end of the beam. The beam is divided into number of equal elements having length L_e.

Where

$b = \sqrt{Da - a^2}$,

$h = \sqrt{D^2 - 4x^2}$ and

$\alpha = \frac{1}{2}\left[\sqrt{D^2 - 4x^2} - (D - 2a)\right]$

The additional strain energy due to the existence of the crack can be expressed as [16], [17].

$$\Pi_c = \int_{Ac} G dA$$

Where G is the strain energy release rate function. The strain energy release rate function can be expressed as

$$G = \frac{1}{E'}\left[(K_{I2} + K_{I3})^2 + K_{II2}{}^2\right]$$

Where $E' = E$ for plane stress problem, $E' = E/1-\mu^2$ for plane strain problem. K_{I2}, K_{I3}, K_{II2} are the stress intensity factors. The values of stress intensity factor can be can be expressed as $K_{ni} = \sigma_i \sqrt{\pi\alpha} F_n\left(\frac{\alpha}{h}\right)$

Using Paris equation

$$\delta_i = \frac{\partial \Pi_c}{\partial P_i} \quad (i = 2,3)$$

The overall additional flexibility matrix C_i

$$C_{ij} = \frac{\partial \delta_i}{\partial P_j} = \frac{\partial^2 \Pi_c}{\partial P_i \partial P_j} \quad (i = 2,3)$$

C_{ij} can be obtained as [25]

$$C_{ij} = \frac{1}{E'} \frac{\partial^2}{\partial P_i P_j} \int_{-\sqrt{(Da-a^2)}}^{\sqrt{(Da-a^2)}} \int_0^{\frac{1}{2}\left[\sqrt{(Da-4x^2)}-(D-2a)\right]} \left[\left\{\frac{32 P_2 L_c h}{\pi D^4}\sqrt{\pi\alpha} F_2\left(\frac{\alpha}{h}\right) + \frac{32 P_3 h}{\pi D^4}\sqrt{\pi\alpha} F_2\left(\frac{\alpha}{h}\right)\right\}^2 + \frac{16 P_2{}^2}{\pi^2 D^4} \pi\alpha F_{II}{}^2\left(\frac{\alpha}{h}\right)\right] d\alpha dx$$

2.1. Overall additional flexibility matrix under conventional fem co-ordinate system

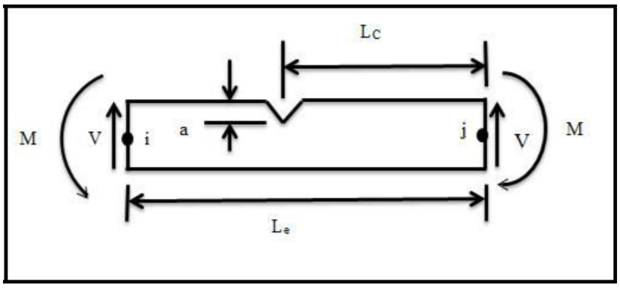

Fig.2. Cracked Timoshenko beam element

Fig.2 shows a cracked beam element with generated loading. The beam is subjected to shearing force *V* and bending moment *M* at each node. The corresponding displacements are denoted as *y* and *θ*. L_c denotes the distance between the right hand side end node and the crack location. *a* denotes the crack depth. The beam element has length L_e, cross-sectional area *A* and flexural rigidity *EI*. Under the FEM co-ordinate and notation system, the relationship between the displacement and the forces can be expressed as

$$\begin{Bmatrix} y_j - y_i - \theta_i \\ \theta_j - \theta_i \end{Bmatrix} = C_{ovl} \begin{Bmatrix} V_j \\ M_j \end{Bmatrix}$$

The flexibility matrix C_{intact} of the intact Timoshenko beam element can be written as

$$\begin{Bmatrix} y_j - y_i - \theta_i \\ \theta_j - \theta_i \end{Bmatrix} = C_{\text{int}act} \begin{Bmatrix} V_j \\ M_j \end{Bmatrix}$$

The Total flexibility matrix of the cracked Timoshenko beam element is obtained by the combination of over-all additional flexibility matrix and flexibility matrix of an intact beam

$$C_{total} = C_{ovl} + C_{\text{int}act}$$

Through the equilibrium conditions, the stiffness matrix „K_c" of a cracked beam element can be obtained as follows [16] [17]

$$K_c = LC_{total}^{-1} L^T$$

Where

$$L = \begin{bmatrix} -1 & 0 \\ -L_e & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3. Result and Discussion

For numerical analysis a simply supported beam with a transverse crack is taken into consideration. The various geometric and physical properties of the beam are *L*=1m, *D*=2cm, *E*=206Gpa, ρ=7800Kg/m^3& μ=0.3.The present result for natural frequencies of uncracked Timoshenko beam is compared with the theoretical result as well as the result obtained by [29].The natural frequencies of cracked beam with various positions and various relative crack depth ratio are also found.

Table 1 Natural Frequency of the beam

ω (rad/sec)	Present Result	Theoretical [29]
ω_1	253.425	253.426

ω_2	1013.703	1013.703
ω_3	2280.835	2280.835

Table 2 Natural Frequencies of a cracked beam, XC/L=0.20

ω (rad/sec)	Xc/L	α/D=0.2	α/D=0.3	α/D =0.4	α/D =0.5
ω_1	0.2	252.05	250.71	249.37	248.27
ω_2	0.2	1013.39	1012.55	1010.95	1008.85
ω_3	0.2	2183.37	1944.38	1700.31	1552.28

Table 3 Natural Frequencies of a cracked beam, XC/L=0.50

ω (rad/sec)	Xc/L	α/D=0.2	α/D=0.3	α/D =0.4	α/D =0.5
ω1	0.5	252.77	252.10	251.38	250.78
ω_2	0.5	982.06	953.60	927.417	907.85
ω_3	0.5	2284.87	2281.5	2268.06	2248.61

Table 4 Natural Frequencies of a cracked beam, XC/L=0.60

ω (rad/sec)	Xc/L	α/D=0.2	α/D=0.3	α/D =0.4	α/D =0.5
ω_1	0.6	251.22	249.08	246.96	245.24
ω_2	0.6	1018.71	1024.41	1031.02	1037.08
ω_3	0.6	2162.82	2058.45	1966.40	1900.87

Table 5 Natural Frequencies of a cracked beam, XC/L=0.80

ω (rad/sec)	Xc/L	α/D=0.2	α/D=0.3	α/D =0.4	α/D =0.5
ω_1	0.8	250.54	247.81	245.13	242.99
ω_2	08	1001.11	989.081	977.32	968.03
ω_3	0.8	2321.40	2350.61	2369.13	2377.00

Table 1 represents the comparison of first three natural frequencies of uncracked Timoshenko beam with theoretical values. Table 3-5 represents the natural frequencies of a cracked Timoshenko beam at various crack positions and relative crack depths. It is observed that natural frequency decreases as crack depth increase. Fig. 3-5 show the mode shapes for uncracked and cracked Timoshenko beam with relative crack position of 0.5 and relative crack depth of 0.2. It is observed that for cracked beam there is a jump at the respective position of crack.

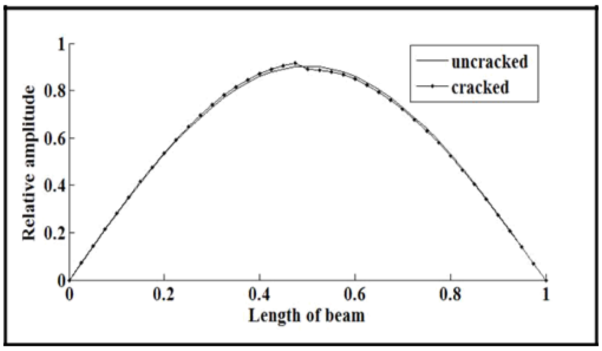

Fig.3 First mode shape for uncracked and cracked beam, $X_C/L=0.5, \alpha/D=0.2$

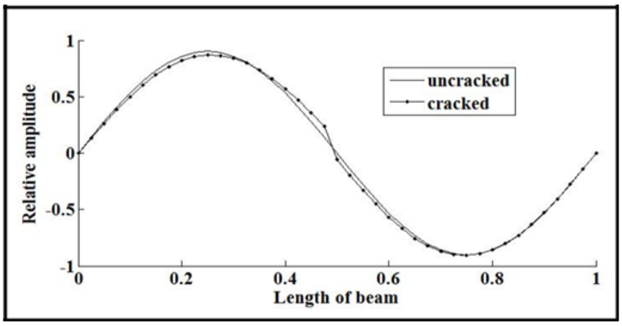

Fig. 4 Second mode shape for uncracked and Cracked beam, $X_C/L=0.5, \alpha/D=0.2$

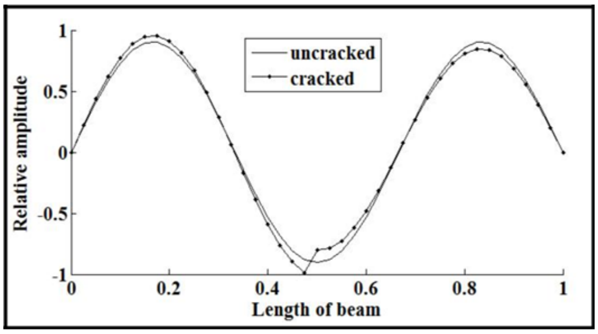

Fig. 5 Third mode shape for uncracked and cracked beam, $X_C/L=0.5, \alpha/D=0.2$

4. Conclusion

In this paper an overall additional flexibility matrix is used for evaluating the natural frequencies of a cracked Timoshenko beam. From the results obtained it has been observed that the presence of crack in the beam decreases the natural frequencies. The magnitude for the change of natural frequencies depends upon the change of relative crack depth and location for the crack. It is observed that due to presence of crack in different positions of the beam the deflection increases.

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