VIBRATION ANALYSIS OF A CRACKED TIMOSHENKO BEAM USING FINITE ELEMENT METHOD

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Abstract: The present day structures and machineries are designed based on maximum strength, longer life, minimum weight and low cost etc., which allow development of a high level of stresses in them which further leads to the development of crack in their elements. Many engineering structures may have structural defects such as cracks due to long-term service, mechanical vibrations, applied cyclic loads etc. So it is very much essential to know property of structures and response of such structures in various cases. The presence of crack in a structure causes a local variation in the stiffness which affects the mechanical behaviour of the entire structure to a considerable extent. Due to the existence of such cracks the frequencies of natural vibration, amplitudes of forced vibration, and areas of dynamic stability change. In order to identify the magnitude and location of the crack, analysis of these changes is essential. The information from the analysis enables one to determine the degree of sustainability of the structural element and the whole structure. Generally for the observation proposes the beam is modeled by two types. Euler-Bernoulli's beam model where only translation mass & bending stiffness have been considered and Timoshenko beam model where both the rotary inertia and transverse shear deformation have been considered. In this paper, the presence of transverse and open crack in the Timoshenko beam has been considered and natural frequencies and mode shapes of the cracked Timoshenko beam have been studied by using finite element method (FEM) and MATLAB programme.

Keywords*:* Crack; Timoshenko beam ;Natural Frequency;Mode Shapes;Finite element method

1. Introduction

For the last several years, extensive research work has been commenced to investigate the faults in structures. It has been observed that most of the structural members fail due to the presence of cracks. Beams are one of the most commonly used elements in structures and machines, and fatigue cracks are the main cause of beams failure. The effect of crack on the dynamic behaviour of the structural elements has been the subject of several investigations for the last few decades. Papadopoulos and Dimarogonas [1] revealed the introduction of local flexibility due to presence of transverse crack in a structural member whose dimension depends on the no of degrees of freedom considered. It has been observed that the local flexibility matrix is mainly appropriate for the analysis of a cracked beam if one employs an analytical method by solving the differential equations piece wisely [2]. It is also appropriate to use a semi-analytical method by using the modified Fourier series [3, 4] and mechanical impedance method [5]. Direct and perturbative solutions for the natural frequencies for bending vibrations of cracked Timoshenko beams are provided by Loyaa [6]. Dynamic stiffness method [7], direct and inverse methods on free vibration of simply supported beams with a crack was derived by Hai-Ping Lin [8]. Crack localization and sizing in a beam from the free and forced response measurements method is indicated by Karthikeyan et al. [9].Identification of cracking in beam structures using Timoshenko and Euler beam formulation has been studied by Swamidas et al. [10]. In their work Timoshenko and Euler beam formulations have been used to estimate the influence of crack size and location on natural frequencies of cracked beam. Frequency contour method has been used to identify the crack size and location properly. Ali et al. [11] have studied the free vibration analysis of a cantilever beam. They have observed that the presence of crack in the beam, affects the natural frequency. The magnitude for the change of natural frequency depends on the change of number, depth and location of the crack. Also the change of dynamic property effects on stiffness and dynamic behaviour. The lowest four natural frequencies of the cracked structure have developed by Shin et al. [12] by using finite element method. They have obtained the approximate crack location by using Armon's Rank-ordering method that uses the above four natural frequencies. A method for shaft crack detection have proposed by Xiang et al. [13] which is based on combination of wave-let based elements and genetic algorithm. Owolabi et al. [14] have investigated experimentally the effects of cracks and damages on the structures .Al-Qaisia et al. [15] have utilized the reduction of Eigen frequencies and sensitivity analysis to localize a crack in a non-rotating shaft coupled to an elastic foundation. The modeling of shaft was done by finite element method and coupled to an experimentally identified foundation model. Sahin et al. [16] have introduced in their research different damage scenarios by reducing the local thickness of the selected elements at different locations along finite element model (FEM) for quantification and localization of damage in beam-like structures. Nahvi et al. [17] have developed an analytical, as well as experimental approach to the crack detection in cantilever beams by vibration analysis. Behzad et al. [18] have calculated natural frequencies for a beam with open edge crack using theoretical and finite element analysis. An investigation of a beam crack identification method by using the standard finite element formulation has been produced by Demos the nous et al. [19]. The finite element analysis of a cracked cantilever beam and the relation between the modal natural frequencies with crack depth, modal natural frequency with crack location has been studied by Sutar [20]. Only single crack at different depth and at different location are evaluated. The analysis reveals a relationship between crack depth and modal natural frequency. Zheng [21] has described an overall flexibility matrix instead of local flexibility matrix in order to find out the total flexibility matrix and the stiffness matrix of the cracked beam. It has been observed that the consideration of 'overall additional flexibility matrix', due to the presence of the crack, can indeed give more accurate results than those obtained from using the local flexibility matrix. The overall additional flexibility matrix parameters are computed by 128-point (1D) or 128 ×128(2D) Gauss quadrature and then further best fitted using the least-squares method. The best-fitted formulas agree very well with the numerical integration results [22]. After getting the stiffness matrix of a cracked beam element standard FEM procedure can be followed, which will lead to a generalized eigenvalue problem and thus the natural frequencies can be obtained.

2. Mathematical Formulation

2.1 *Finite element modeling of regular beam element*

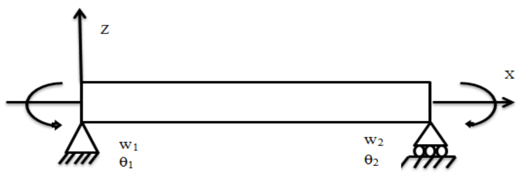

Fig-1.A regular simply supported Timoshenko beam element

The regular beam element is shown in fig-1 .The longitudinal axis of the regular beam element lies along the X-axis. The element has constant moment of inertia, modulus of elasticity, mass density and length [23].The displacement relation in the x, y and z directions of the beam is given by

$$u(x,y,z,t) = z\theta(x,t) = z\left(\frac{\partial w}{\partial x} - \beta(x)\right)$$

$$v(x,y,z,t) = 0\,,\, w(x,y,z,t) = w(x,t).$$

Where the axial displacement along the x-axis *is* u, the lateral displacement along the *y* axis is v which is equal to zero and the time dependent transverse displacement of the centroidal axis (along *z* axis) is w. The time dependent rotation of the cross-section about *y* axis is θ. The shear *is β(x)* which has no contribution during finding out the axial displacement at a point at a distance z from the centre line. The equation of motion can be derived by using Hamilton's principle as the total strain energy being equal to the sum of change in kinetic energy and the work done due to the external forces and is given by

$$\delta\Pi = \int_{t_1}^{t_2} (\delta U - \delta T - \delta W_e)dt = 0.$$

Here U is the strain energy, T is the kinetic energy and W_e is the external work done which is equal to zero and t is the time.Sustituting the values of strain energy and kinetic energy, we get the governing equation of motion

(Timoshenko beam equation) of a general shaped beam modelled with Timoshenko beam theories. Let w be the approximated by a cubic polynomial and is given as

$$w = a_1 + a_2 x + a_3 x^2 + a_4 x^3 .$$

Where x is the distance of the finite element node from the fixed end of the beam.a_1, a_2 a_3 *and* $a4$ are the unknown coefficients and are found out by using boundary conditions at the beam ends. The mass matrix of the regular beam element is the sum of the translational mass and the rotational mass and is given in matrix form as

$$[M_b] = \int_0^{L_e} \begin{bmatrix} [N_w] \\ [N_\theta] \end{bmatrix}^T \begin{bmatrix} \rho A & 0 \\ 0 & \rho I \end{bmatrix} \begin{bmatrix} [N_w] \\ [N_\theta] \end{bmatrix} dx .$$

The stiffness matrix of the regular beam element is the sum of the bending stiffness and the shear stiffness and is written in matrix form as

$$[K_b] = \int_0^{L_e} \begin{bmatrix} \dfrac{\partial [N_\theta]}{\partial x} \\ [N_\theta] + \dfrac{\partial [N_w]}{\partial x} \end{bmatrix}^T \begin{bmatrix} EI & 0 \\ 0 & kGA \end{bmatrix} \begin{bmatrix} \dfrac{\partial [N_\theta]}{\partial x} \\ [N_\theta] + \dfrac{\partial [N_w]}{\partial x} \end{bmatrix} dx .$$

Where $[N_w]$ & $[N_\theta]$ are the shape functions for displacement and rotation, taking the shear (ϕ) into consideration

$$\phi = \frac{12EI\, k_y}{AG\, L^2} , \ \frac{1}{k} = k_y .$$

Substituting the mode shapes $[N_w]$ & $[N_\theta]$ we get the stiffness matrix of the regular beam element. When ϕ is neglected, the mass matrix and the stiffness matrix reduce to the mass and stiffness matrix of an Euler-Bernoulli beam.

2.2 Finite element modelling of a cracked beam element

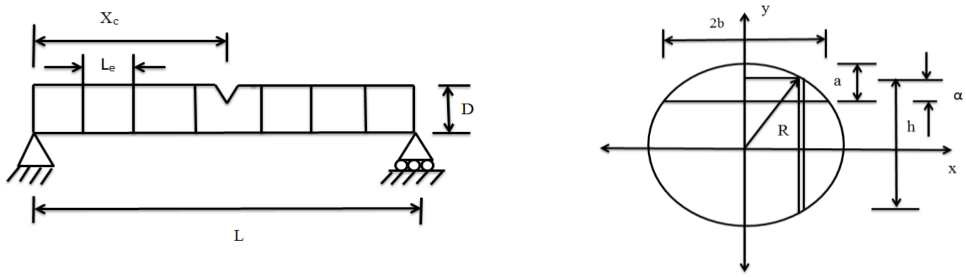

Fig.2 Simply supported beam with crack subjected to shear force and bending moment

A simply supported beam of circular cross-section having diameter 'D' with a single transverse crack with constant depth 'a' is shown in Fig.2. The crack is at a distance of 'X_c' from the left end of the beam. The beam is divided into number of equal elements having length 'Le'.

Where $b = \sqrt{Da - a^2}$, $h = \sqrt{D^2 - 4x^2}$ and $\alpha = \frac{1}{2}\left[\sqrt{D^2 - 4x^2} - (D - 2a)\right]$

2.2.1 Overall additional flexibility matrix under conventional FEM co-ordinate system:

A cracked beam element subjected to shearing force 'V' and bending moment 'M' at each node is considered. The corresponding displacements are denoted as 'y' and 'θ'. 'Lc' denotes the distance between the right hand side end node and the crack location. 'a' denotes the crack depth. The beam element has length 'Le', cross-sectional area is A and flexural rigidity EI. Under the FEM co-ordinate and notation system, the relationship between the displacement and the forces can be expressed as

$$\begin{Bmatrix} y_j - y_i - \theta_i \\ \theta_j - \theta_i \end{Bmatrix} = C_{ovl} \begin{Bmatrix} V_j \\ M_j \end{Bmatrix} .$$

2.2.2 Flexibility matrix for intact beam element:

The flexibility matrix C_{intact} of the intact Timoshenko beam element can be written as

$$\begin{Bmatrix} y_j - y_i - \theta_i \\ \theta_j - \theta_i \end{Bmatrix} = C_{\text{intact}} \begin{Bmatrix} V_j \\ M_j \end{Bmatrix}.$$

2.2.3 Total flexibility matrix of the cracked Timoshenko beam element:

The total flexibility matrix of the cracked Timoshenko beam element is obtained as

$$C_{total} = C_{ovl} + C_{\text{intact}}.$$

2.2.4 Stiffness matrix of a cracked Timoshenko beam element:

Through the equilibrium conditions, the stiffness matrix 'Kc' of a cracked beam element can be obtained as follows [27, 28]

$$K_c = L C_{total}^{-1} L^T.$$

Where L is the transformation matrix.

3. Result and Discussion

A simply supported Timoshenko beam with a transverse crack is considered for numerical analysis. The various geometric and physical parameters of the beam are L=1m, D=2cm, E=206Gpa, ρ=7800Kg/m^3& μ=0.3.The beam length is divided into forty no of elements. The natural frequencies for the uncracked and cracked Timoshenko beam are shown in Table 1-5. The first three mode shapes of uncracked and cracked Timoshenko beam are shown in Fig.3 (a) to 3(c).

Table-1.Natural frequencies for uncracked Timoshenko beam.

ω (rad/sec)	Present [FEM]	Theoretical [29].
ω_1	253.425	253.426
ω_2	1013.703	1013.703
ω_3	2280.835	2280.835

Table-2. Natural Frequencies of Cracked Timoshenko beam, Xc/L=0.20.

ω (rad/sec)	Xc/L	α/D=0.2	α/D =0.3	α/D =0.4	α/D =0.5
ω_1	0.2	252.053	250.716	249.371	248.275
ω_2	0.2	1013.396	1012.553	1010.952	1008.875
ω_3	0.2	2183.378	1944.381	1700.315	1552.258

Table-3. Natural Frequencies of Cracked Timoshenko beam, Xc/L=0.50.

ω (rad/sec)	Xc/L	α/D=0.2	α/D =0.3	α/D =0.4	α/D =0.5
ω_1	0.5	252.772	252.100	251.389	250.784
ω_2	0.5	982.0696	953.609	927.4137	907.8555
ω_3	0.5	2284.857	2281.51	2268.067	2248.615

Table-4. Natural Frequencies of Cracked Timoshenko beam, Xc/L=0.60.

ω (rad/sec)	Xc/L	α/D=0.2	α/D =0.3	α/D =0.4	α/D =0.5
ω_1	0.6	251.221	249.088	246.960	245.240
ω_2	0.6	1018.711	1024.418	1031.028	1037.089
ω_3	0.6	2162.823	2058.453	1966.404	1900.875

Table-5. Natural Frequencies of Cracked Timoshenko beam, Xc/L=0.80.

ω (rad/sec)	Xc/L	α/D=0.2	α/D =0.3	α/D =0.4	α/D =0.5
ω_1	0.8	250.544	247.810	245.130	242.997
ω_2	0.8	1001.11	989.0831	977.321	968.032
ω_3	0.8	2321.405	2350.619	369.132	2377.006

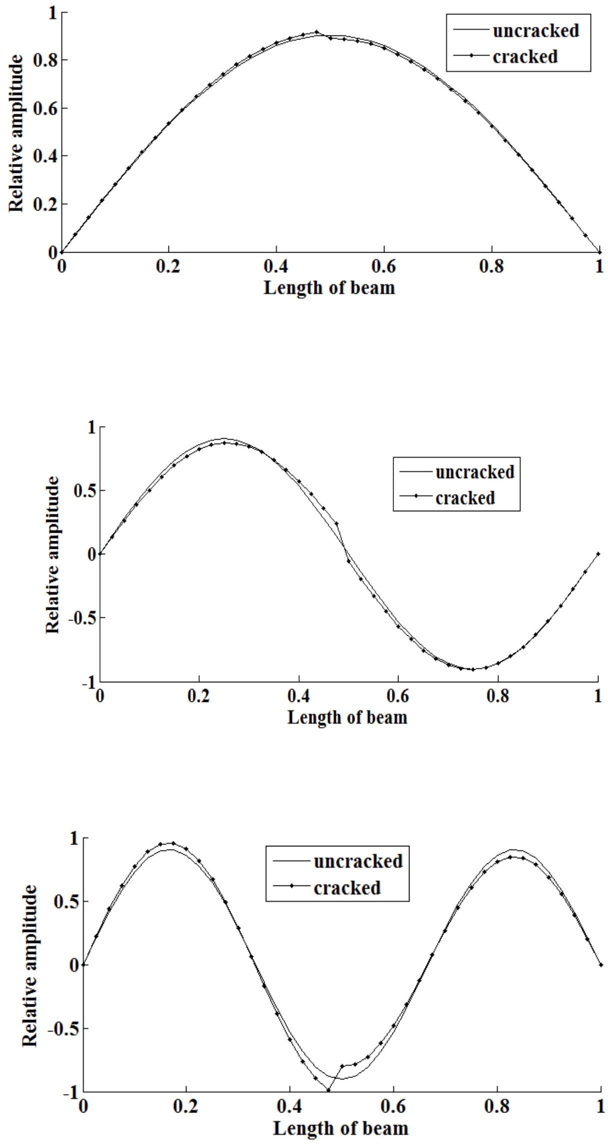

Fig.3 (a) First mode shape (b) Second mode shape and (c) third mode shape for uncracked and cracked beam, $X_C/L=0.5,\alpha/D=0.2$

4. Conclusion

In this work overall additional flexibility matrix is considered for evaluating the natural frequencies of a cracked Timoshenko beam. From the results obtained it has been observed that the presence of crack in the beam decreases the natural frequencies. The magnitude for the change of natural frequencies depends upon the change of relative crack depth and location for the crack. It is observed that with increase in relative crack depth the natural frequency decreases and there is an abrupt change in mode shape due to presence of crack.

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